Journal of Sustainable Development Issues, 2026, 4(1),
pp. 121-144.
Journal homepage: https://journalsdi.com/index.php/jsdi/index; ISSN: 3078-2546

Oil Price Shocks and CO2 Emissions in Azerbaijan: Evidence from a Multiple Threshold Nonlinear ARDL Approach

Nijat Gasim[1,*], Nazila Zahidova[2], Farhad Mirzayev[3]

[1] *Department of Biostatistics, Karabakh University, Khankendi, Azerbaijan*
[2] *Department of Algebra and Geometry, Azerbaijan State Pedagogical University, Baku, Azerbaijan*
[3] *Department of Economic Cybernetics, Baku State University, Baku, Azerbaijan*

ABSTRACT

This study investigates the regime-dependent effect of Brent petroleum price shocks on CO_2 emissions in Azerbaijan for the period of 1990-2024. Azerbaijan is an oil-exporting, resource-dependent economy, so that changes in Brent prices may influence emissions through oil revenues, fiscal expansion, investment activity, energy use and energy-intensive production. Unlike prior research that investigates factors affecting CO_2 emissions and the macroeconomic impacts of oil prices separately, this paper directly analyzes the nexus between Brent oil price shocks and CO_2 emissions within a Multiple Threshold Nonlinear ARDL framework. First, Harvey-type linearity tests show that LCO_2 and LBPP are nonlinear. The unit root tests, such as ADF, Flexible Fourier ADF and Fractional Flexible Fourier ADF tests, reveal that the variables are integrated of order one, and are suitable for ARDL-type modeling. The results of MTNARDL confirm the long-run relationship between the Brent price shock regimes and CO_2 emissions. The error correction coefficient is significant and negative implying a rapid adjustment towards the long-run equilibrium. The effects in the long-run are mostly observed in the Q1 and Q2 regimes, while the effects in the short-run are concentrated in the Q3 and Q4 regimes. The strongest long-run response is observed in the Q2 regime, suggesting that weak or transitional changes in Brent prices are particularly relevant to the long-run behavior of CO_2 emissions. Further Wald asymmetry tests corroborate that the relationship is regime-dependent and not linear and uniform. The results show that the environmental consequences of oil price shocks in Azerbaijan depend on the size of the shock and the time horizon considered.

Keywords: Brent petroleum price, CO_2 emissions, Azerbaijan, MTNARDL, Oil price shocks, Regime dependence.

* Corresponding author: Department of Biostatistics, Karabakh University, Khankendi, Azerbaijan
nijat.gasim@karabakh.edu.az

JEL Classification: C22, C12, Q35, Q53
DOI: 10.62433/josdi.v4i1.83

"This journal is licensed under a Creative Commons Attribution 4.0 Interntional license."

Received: December, 2025 | 1st Revision: February, 2026 | Accepted: May, 2026

1. INTRODUCTION

The increase in carbon emissions is considered as one of the biggest global challenges of our time with respect to climate change, energy security and sustainable economic development. Industrialization, increase in energy consumption and the economic activities based on fossil fuels are the main causes of the carbon dioxide emissions (Kartal et al., 2024; Kartal et al., 2025). This problem is especially complicated in countries with abundant oil and gas resources. The reason is that in such economies energy resources are not only an important factor in the production and consumption process, but also one of the main determinants of government revenue, exports, investment and macroeconomic stability. Thus, oil price fluctuations should be seen not only as volatility in the price of energy but as external macro-economic shocks that shape economic activity and environmental outcomes in resource-dependent economies.

The economic and environmental implications of international oil prices need to be given more attention in oil-exporting countries. Higher oil prices can increase export revenues, fiscal capacity and economic activity but on the other hand they may increase carbon emissions by expanding energy-intensive production, transportation, construction and industrial activities. An increase in crude oil prices leads to an increase in energy consumption and CO_2 emissions in the short and long term, as shown in a case study of Ecuador by Nwani (2017). The author argues that high oil prices can increase energy use and emissions in oil-dependent economies through the income effect, energy consumption subsidies, and the expansion of energy-intensive sectors. Such an approach implies that oil prices have significant effects on economic growth and environmental quality.

In this regard, Azerbaijan is a unique case study. The country has undergone post-Soviet transformation, is an oil and gas exporter and heavily depends on hydrocarbon resources. In terms of the economy of Azerbaijan, oil prices are among the main external determinants of macroeconomic dynamics. The relationship between global oil prices and GDP in Azerbaijan was investigated by Ayyubova (2023) using the VECM approach. She demonstrated that Azeri Light, Brent and WTI prices have long-term reciprocal relationship with the macroeconomic indicators of the country. These results confirm that oil prices are not only a factor of export revenues for the economy of Azerbaijan but also a major external shock in terms of economic activity, investment, fiscal revenues and overall macroeconomic stability.

Various determinant's studies on CO_2 emissions in Azerbaijan have been carried out. The impact of economic growth, energy consumption, total factor productivity, international trade, tourism, and financial development on CO_2 emissions has been studied in previous studies. Hasanov et al. (2023) analyzed consumption-based CO_2 emissions in Azerbaijan in the context of total factor productivity, renewable energy consumption, income, exports, and imports. Total factor productivity and renewable energy consumption variables were found to decrease emissions while income and imports variables increased emissions. The effect of renewable energy consumption variable on consumption-based CO_2 emissions is also found to be negative in case of Azerbaijan by Mukhtarov et al. (2023).

Nevertheless, there is a large lacuna in the existing literature on Azerbaijan. On the one hand, the macroeconomic effects of oil prices on the Azerbaijani economy have been studied and, on the other hand, the different determinants of CO_2 emissions have been analyzed separately. However, the direct effect of Brent oil prices on CO_2 emissions in Azerbaijan has not been systematically explored yet, particularly regime-specific with regard to the scale of oil price shocks. In other words, while the existing literature separately studies the "oil price-macroeconomy" nexus and the determinants of CO_2 emissions, the "Brent oil price shocks-CO_2 emissions" link that links these two strands of research remains under-explored for Azerbaijan. The present study attempts to fill this gap by exploring the regime-dependent effect of Brent oil price shocks on CO_2 emissions using the MTNARDL approach.

The main objective of this study is to estimate the effects of Brent petroleum price shocks on CO_2 emissions in Azerbaijan via Multiple Threshold Nonlinear ARDL approach. The central research question is stated as: Does the impact of low, medium and high Brent oil price shocks on CO_2

emissions in Azerbaijan differ? For an economy like Azerbaijan, which exports oil, this question is important both theoretically and politically. If high shocks in the Brent price increase CO_2 emissions more strongly, this may suggest that oil revenues stimulate carbon emissions in addition to economic activity. If the effect only occurs at certain threshold levels this would indicate that the relationship between oil price and emissions is regime-dependent.

Our contribution to the literature has two main aspects. First, this study introduces the variable "Brent oil price" into the literature on the determinants of CO_2 emissions in Azerbaijan, treating it as a direct external oil price shock. Second, the study examines the positive and negative directions of Brent price shocks, with the low, medium and high shock regimes, to shed light on the regime-dependent nature of the oil prices –emissions relationship.

The remainder of the study is organized as follows. Section 2 analyzes the literature on oil price shocks, resource-dependent economies, energy transition, and CO_2 emissions, with specific focus on evidence from Azerbaijan and nonlinear threshold-based techniques. Section 3 describes the technique, including the baseline model formulation and the econometric strategy based on MTNARDL. In section 4, we present the data and empirical results which include descriptive statistics, graphical analysis, linearity tests, unit root tests, MTNARDL estimations, bounds test results, diagnostic tests, dynamic multipliers and asymmetry analysis. The key findings are presented in section 5 and related to the current literature. Section 6 presents policy recommendations and finishes the paper with a summary of the key findings, limitations and possible direction for future research.

2. LITERATURE REVIEW

2.1. The Relationship Between Oil Price Shocks and Carbon Emissions

In addition to being one of the most important indicators of the global energy markets, the price of oil appears as one of the key external determinants of macroeconomic activity, fiscal revenues, investment behaviour and energy demand in oil-exporting economies. The price of Brent crude oil can be viewed as an external macroeconomic shock that can have an impact on carbon emissions in resource-dependent economies via the channels of production, consumption, government expenditure and energy consumption, in addition to being a price signal set in the international energy market in this context.

The link between oil prices and CO_2 emissions is particularly intricate in oil exporting countries. Higher oil prices contribute to export revenues and the fiscal capacity of the government. Such an increase can lead to an expansion of energy-intensive activities such as government spending, infrastructure projects, industrial production, transportation, and construction. Thus, an increase in oil prices causes an increase in economic activity and in energy consumption, which results in an increase of CO_2 emissions. Conversely, increasing oil prices may, in certain situations, generate incentives for energy efficiency and switch to alternative energy sources. Thus, the impact of oil prices on emissions may vary depending on a country's energy mix, fiscal policy, subsidies, economic diversification, and energy transition strategy.

One of the closest empirical studies along this line was done by Nwani (2017) on the Ecuador sample. The relationship between crude oil price, energy consumption, economic growth and CO_2 emissions for the period 1971–2013 is investigated by the author using ARDL and Toda–Yamamoto (1995) causality test. The results have shown that crude oil price increase has increased energy consumption and CO_2 emissions in short and long term. Furthermore, the causal relationship between crude oil price and energy consumption and economic growth has been established. This result implies that high oil prices in oil dependent economies may increase emissions through economic activity and energy use.

Nwani (2017) explains three main channels that allows the oil price and emission are related. The first channel is the income effect. Higher crude oil prices increase the revenues of governments and households in oil-exporting countries, which can lead to an increase in the demand for energy-intensive technologies and products. The second channel is subsidies to energy. Direct or indirect subsidization of energy consumption by governments during periods of high oil revenues leads to increased energy waste and CO_2 emissions. The third channel is the expansion of energy-intensive sectors. The more fossil fuel is used, the worse the environmental quality becomes as oil revenues stimulate activity in sectors such as industry, transportation and infrastructure. This mechanism is also theoretically very relevant for economies like Azerbaijan's which are sensitive to oil and gas revenues.

Moreover, the relationship between oil prices and energy consumption and environmental indicators has been studied in other oil exporting countries. For instance, Agbanike et al. (2019) pointed out that the nexus between crude oil prices, energy consumption and environmental indicators is indirect for oil-dependent economies and can be affected by economic growth, government spending and trade channels. This approach suggests that in some cases the crude oil price may not have a direct impact on the CO_2 emissions but can have an indirect impact via its influence on the economic activity and the energy demand . Thus, the effect of oil price shocks on carbon emissions needs to be considered both directly and indirectly, considering macroeconomic mechanisms.

2.2. Resource-Based Economies, The Energy Transition, and Carbon Lock-in

The environmental impacts of high oil prices in oil-exporting countries are not limited to higher emissions. Such an impact can also impact the pace of energy transition, energy investment flows and the sustainability of a fossil fuel-based development model. The “carbon lock-in” approach is an important theoretical framework in this context. The concept of carbon lock-in is that the economic technological, institutional and investment structures are still fossil fuel based. High oil prices make the fossil fuel sector more economically attractive, which can slow the renewable energy transition and make carbon-intensive economic activities more realistic.

This mechanism is empirically proved by Mukhtarov et al. (2022) by the Iranian case study. The authors used a General-to-Specific modeling approach to examine the relationship of oil prices, CO_2 emissions, income and renewable energy consumption for the period 1980–2019. The results showed that oil prices and CO_2 emissions had a statistically significant negative effect on the consumption of renewable energy (Mukhtarov et al., 2022). This result was explained by the authors as the “higher oil price satisfaction effect”. Higher oil prices, in other words, increase revenues from conventional energy sources in oil-exporting countries and as such may reduce the incentive for the renewable energy transition process.

Low-carbon energy sources are acknowledged in the energy transition literature as important tools for reducing CO_2 emissions. Aliyev et al. (2024) investigated the impact of nuclear energy consumption on CO_2 emissions in South Korea through Fourier-based econometric approaches. Their results indicate that nuclear energy consumption reduces CO_2 emissions in the long-run and that low-carbon energy sources can have a significant role in improving the environmental quality. This evidence is relevant for the present study as it shows that the environmental impact of energy-related-variables depends not only on the amount of energy used but also on the structure of the energy system and the transition towards low-carbon sources .

The result is particularly important for Azerbaijan. Azerbaijan is an oil and gas exporter and has been working in recent years to increase its utilization of its revitalized energy potential. But a return to a fossil fuel-based economic model may be tempting when high Brent prices boost oil revenues. In this context, the Brent price is not only an external price shock that affects CO_2 emissions, but also a strategic factor that alters the economic incentives of the energy transition. Therefore, the relationship between Brent oil price and CO_2 is of great importance from both environmental and energy transition policy point of view.

Another relevant strand of the literature highlights the importance of technological innovation and digital transformation for environmental outcomes. Mukhtarov et al. (2026) studied the environmental impact of AI-led innovation in the United States. Although their study focuses on a technologically advanced economy rather than an oil exporting country, it is relevant to the present research as it suggests that innovation, digitalization and technology-oriented transformation may play an important role in shaping environmental quality. This perspective is important for resource-dependent economies like Azerbaijan as the environmental impacts of oil price shocks can also be affected by the fact that additional oil revenues are allocated to traditional fossil fuel-based activities or to cleaner technologies, energy efficiency and innovation-led development.

The impact of oil prices on the energy transition and low-carbon assets has also been investigated in financial markets. The RALS cointegration approach was applied by Şahinler (2025) to analyze the long-run relationship between oil prices, carbon allowance prices, the technology index and low-carbon footprint stocks. The findings showed that the oil price variable has a positive impact on low-carbon footprint stocks. The result implies that high oil prices may induce interest in alternative energy and low-carbon investments in some markets. However, this mechanism may be working differently in oil-exporting countries. In these countries, high oil prices may sometimes reinforce the fiscal and economic role of the fossil fuel sector, rather than accelerating the energy transition. The national context is thus relevant for understanding the relationship between oil prices and environmental outcomes.

2.3. Literature on Oil Prices and CO_2 Emissions in Azerbaijan

The literature on Azerbaijan has studied specifically the impact of oil prices on macro-economic indicators. Ayyubova (2023) analyzed the impact of global oil prices on the GDP of Azerbaijan using the VECM approach. The study investigated the bidirectional relationships between the prices of Azeri Light, Brent, and West Texas Intermediate and Azerbaijan's GDP, showing that these price indicators are strongly linked to the dynamics of the country's economy. The author states that the external factors such as the global oil prices are important for the macroeconomic stability of Azerbaijan, namely, its GDP, investment flows, real exchange rate. This leads to a theoretical and economic justification of the inclusion of the variable of the Brent oil price in the empirical model for Azerbaijan.

On the other hand, there are also studies that analyze the determinants of CO_2 emissions in Azerbaijan. Hasanov et al. (2023) estimated the consumption-based CO_2 emissions in Azerbaijan within the framework of total factor productivity, renewable energy consumption, income, exports and imports. Hasanov et al. (2023) found that the variables TFP and renewable energy consumption reduce CO_2 emissions, while the variables income and imports increase emissions. This finding indicates that the production efficiency, energy structure and international trade channels shape the carbon emissions of Azerbaijan.

Mukhtarov et al.(2023) also investigated the nexus between renewable energy consumption and consumption-based CO_2 emissions in Azerbaijan by employing the DOLS approach. Their results indicate that CO_2 emissions are reduced by renewable energy consumption, whereas real GDP per capita and imports increase CO_2 emissions. Despite an oil-rich economy, the variable of renewable energy consumption can play a mitigating role from an environmental perspective as shown by this study for Azerbaijan. However, the oil price variable was not found to be a key determinant in this study.

Another topic of focus for Azerbaijan concerns carbon emissions related to tourism and trade. Mikayilov et al. (2020) examined the relationship between international tourism and consumption-based CO_2 emissions in Azerbaijan and found an N-shaped relationship between tourism revenues and CO_2 emissions. The results of the study reveal that the tourism-based EKC hypothesis is not valid for Azerbaijan; the import variable increases CO_2 emissions and the export variable decreases CO_2 emissions. This finding draws the need for a separate assessment of the impact of international economic activities on carbon emissions in resource-rich countries in the phase of diversification.

However, while these studies are important in terms of explaining the different determinants of CO_2 emissions in Azerbaijan, none of them has estimated the direct impact of the Brent oil price variable on CO_2 emissions through a multiple-threshold approach. This means that Azerbaijan has research lines of "oil price-macroeconomics" and "CO_2 determinants" separately. However, the relationship of "Brent crude oil price shocks and CO_2 emissions" has not been studied as a separate issue or in a regime-dependent manner. This study will combine these two lines of research and analyze the impact of Brent price shocks on carbon emissions in Azerbaijan.

2.4. Non-linear and multiple-threshold approaches

The effect of oil price shocks on CO_2 emissions may not be linear. Linear models assume that the average effect of all changes in the Brent price is equal. However, the responses of economic activity, energy use and emissions to small, moderate and large oil price shocks may be through different mechanisms. Small price changes may not be enough to change economic behavior. Moderate price changes could affect the activity of some sectors. However, large Brent price shocks may have a more important impact on CO_2 emissions by using channels such as oil revenues, government spending, investment and energy demand. Therefore, it is possible that the ARDL model itself does not adequately represent the real dynamics of this relationship.

The NARDL approach is more flexible than the ARDL model since it allows the estimation of positive and negative changes in the variables separately (Shin et al., 2014). However, the NARDL model considers only two regimes: positive and negative changes. This does not allow full discrimination of the magnitude of the shock. However, Multiple Threshold NARDL model is a generalization of NARDL which allows the explanatory variable to be decomposed into multiple threshold regimes (Pal & Mitra, 2015, 2016). The approach considers not only the direction of the shock but also its magnitude. The present study's division of the Brent crude oil price variable into low, medium and high shock regimes is based exactly on this methodological logic.

In recent years, the MTNARDL approach has been increasingly used in the ecological economics literature. Uche and Effiom (2021) examined the relationship between financial development and CO_2 emissions in Nigeria using the MTNARDL model and found that the effect of major, moderate, and minor changes in the financial development variable differ. The authors observed that the MTNARDL model gives more robust results than the NARDL model and is better able to capture the differences in effects based on the size of changes in the explanatory variables. This finding supports the use of the MTNARDL approach in analyzing unpredictable and shock-prone indicators like Brent price shocks.

Asif et al. (2023) conducted a comparative analysis of the relationship between fiscal deficit and CO_2 emissions in India using the ARDL, NARDL and MTNARDL approaches. The findings of the research show that the impact of the fiscal deficit variable on CO_2 is asymmetric and the MTNARDL model more distinctly displays different impacts for extreme low, moderate and extreme high changes. This study reveals that the macroeconomic shocks' impact on CO_2 emissions could be conditional not only on the shock direction but also on the magnitude of the shock.

On the other hand, Ben-Salha and Ayad (2025) have evaluated the effect of climate policy uncertainty on aggregate and sectoral CO_2 emissions for the United States by applying the MTNARDL approach. The authors showed that the standard ARDL and NARDL models are unable to show some relations but the MTNARDL model can identify the differential effects of small and large changes in the climate policy uncertainty variable on CO_2 emissions. The results show that large changes in climate policy uncertainty have a reducing effect on CO_2 emissions while small changes may in some cases worsen environmental quality. This result shows the power of the multiple-threshold approach to uncover hidden nonlinear effects in ecological models.

This method is a direct basis for the ongoing research in the literature. Brent crude oil prices are highly volatile, and the impact of Brent crude oil prices on oil-exporting economies should be evaluated not only in terms of "increases" and "decreases," but also in terms of "small," "medium," and "large" shocks. Hence, the MTNARDL approach is justified from the theoretical and methodological point of view to model the relationship $LCO_2 = f(LBPP)$ for Azerbaijan.

2.5. The Gap in the Literature and an Overview of Current Studies

A general review of the literature accessible on the market shows that there have been studies on the relationship between oil prices, energy consumption and CO_2 emissions at international level. In particular, studies on oil-dependent economies reveal that crude oil prices can increase CO_2 emissions through various channels such as economic activity and energy consumption (Nwani, 2017). On the other hand, studies on resource-rich countries suggest that high oil prices could prevent the transition to renewable energy (Mukhtarov et al., 2022). These findings indicate that changes in the price of Brent crude oil could be an important environmental contributor for oil-exporting countries like Azerbaijan.

Two main perspectives have arisen in the literature on Azerbaijan. The first one examines the macroeconomic effects of oil prices. In this regard, Ayyubova (2023) investigated the relationship between world oil prices and GDP of Azerbaijan by using the VECM approach. Another line of inquiry is concerned with the determinants of CO_2 emissions. Such strand has included research such as Hasanov et al. (2023), Mukhtarov et al. (2023), and Mikayilov et al. (2020) that have evaluated the effect of factors such as renewable energy, TFP, income, trade, and tourism on CO_2 emissions. However, still there is a research deficit in studies that combine these two methodologies, i.e., that directly examine the effect of Brent crude oil price shocks on CO_2 emissions in Azerbaijan.

Moreover, the impact of the "Brent price shocks" variable on CO_2 emissions in low, medium and high shock regimes has not been evaluated in the Azerbaijani literature. Oil prices are, however, highly volatile and the effect of this volatility on emissions may not be the same in all periods. Small changes in Brent prices may have weak impact on carbon emissions, yet large Brent price shocks could change emissions more significantly through channels such as oil revenues, energy-intensive activities and fiscal expansion. In this sense, the multiple-threshold approach provides new and more appropriate empirical framework for Azerbaijan.

This study seeks to fill that void. This article will evaluate the reaction of CO_2 emissions in Azerbaijan to Brent oil price shocks by MTNARDL approach. The key distinction of this study is that we model Brent prices not as a single-line determinant, but separately for low, medium and high shock regimes. In addition, the study evaluates whether the effect of Brent price shocks on CO_2 emissions differs across negative, weak, moderate positive and large positive shock regimes. In this regard, the study adds a nonlinear and regime-dependent empirical perspective to the literature on oil prices and carbon emissions in Azerbaijan.

As shown in the above literature, the link between oil prices, energy consumption and carbon emissions has been studied by employing various methodological approaches at international and country specific levels. However, the current literature suggests that the impact of oil prices on the environment may vary depending on a country's resource dependence, energy mix, fiscal behavior and energy transition strategy. While the macroeconomic effects of oil prices and determinants of CO_2 emissions have been separately studied for Azerbaijan, the direct and regime-dependent effects of Brent oil price shocks on CO_2 emissions have not yet been sufficiently analyzed. In this context, the main empirical findings from the literature on oil prices, resource-dependent economies and CO_2 emissions are summarized in Table 1.

Table 1. *Main studies on oil prices, resource-dependent economies, and CO_2 emissions*

Author and year	Country / context	Key variables	Method	Key finding	Link to the current study
Nwani (2017)	Ecuador	Crude oil price, energy consumption, CO_2, economic growth	ARDL, Toda–Yamamoto	Crude oil prices increase energy consumption and CO_2 emissions.	Provides a key international benchmark for the Brent price–CO_2 emissions channel.
Agbanike et al. (2019)	Oil-dependent economy	Crude oil price, energy consumption, CO_2, economic activity	ARDL / causality approach	Oil prices may affect CO_2 emissions through economic activity and energy consumption.	Supports the indirect transmission mechanism in oil-dependent economies.
Mukhtarov et al. (2022)	Iran	Oil price, CO_2, income, renewable energy consumption	General-to-Specific modeling	Oil prices have a negative effect on renewable energy consumption.	Suggests that high oil prices may slow down the energy transition process.
Ayyubova (2023)	Azerbaijan	Azeri Light, Brent, WTI, GDP	VECM	World oil prices are associated with Azerbaijan's GDP dynamics.	Justifies Brent price as an external macroeconomic shock for Azerbaijan.
Hasanov et al. (2023)	Azerbaijan	Consumption-based CO_2, TFP, REC, GDP, exports, imports	Autometrics, cointegration	TFP and REC reduce CO_2 emissions, whereas income and imports increase emissions.	Provides evidence on the determinants of CO_2 emissions in Azerbaijan.
Mukhtarov et al. (2023)	Azerbaijan	REC, GDP, exports, imports, consumption-based CO_2	DOLS	REC reduces CO_2 emissions, while GDP and imports increase emissions.	Supports the energy–CO_2 emissions nexus in Azerbaijan.
Mikayilov et al. (2020)	Azerbaijan	Tourism, imports, exports, consumption-based CO_2	Long-run estimation	The tourism–CO_2 relationship is N-shaped; imports increase CO_2 emissions.	Shows the environmental implications of international economic activity in a resource-rich economy.
Şahinler (2025)	European financial markets	Oil prices, carbon allowance, low-carbon stocks	RALS cointegration	Oil prices have a positive effect on low-carbon stocks.	Provides a comparative background on the effect of oil prices on low-carbon assets.

As shown in Table 1, the international literature explains the relationship between crude oil prices and CO_2 emissions mainly through energy consumption, economic activity and resource revenues channels. This channel is directly empirically supported by Nwani (2017) who shows that oil prices increase energy consumption and CO_2 emissions. By contrast, Mukhtarov et al. (2022) argue that high oil prices can be a barrier for the process of transition to renewable energy, as oil prices influence not only the level of emissions but also the motivation for the energy transition. Studies on Azerbaijan emphasize the macroeconomic importance of Brent and other oil prices and different determinants of CO_2 emissions, but they do not specifically analyze the relation between Brent price shocks and CO_2 emissions. This shows clearly how the present study fills a major gap in the literature.

Oil price shocks could potentially linearly and constantly impact carbon emission. For example, the economic and environmental effects of small, medium and large shocks can be different for indicators with high volatility, such as the Brent prices. Therefore, in some cases, the standard ARDL and NARDL models may be unable to capture the latent nonlinear effects arising from the magnitude of shocks. The Multiple Threshold Nonlinear ARDL approach, on the other hand, allows us to evaluate different effects by the magnitude of the shocks when the explanatory variable is divided into a number of threshold regimes. The theoretical foundations of this methodological approach are summarized in Table 2, along with examples of its use in the environmental economics literature.

Table 2. *Methodology and Empirical Literature on the MTNARDL Approach*

Author and year	Context	Methodological approach	Main finding	Link to the current study
Shin et al. (2014)	Methodological	NARDL	Separates the asymmetric effects of positive and negative shocks.	Provides the baseline nonlinear ARDL framework.
Pal and Mitra (2015, 2016)	Methodological / price transmission	MTNARDL	Allows explanatory variables to be decomposed into multiple threshold regimes.	Provides the main methodological basis for modeling Brent price shocks.
Uche and Effiom (2021)	Nigeria	MTNARDL	The effects of major, moderate, and minor changes in financial development differ across regimes.	Highlights the importance of shock size in CO_2 emissions models.
Asif et al. (2023)	India	ARDL, NARDL, MTNARDL	Extreme changes in fiscal deficit have different effects on CO_2 emissions.	Supports the threshold-dependent impact of macroeconomic shocks on CO_2 emissions.
Ben-Salha and Ayad (2025)	United States	MTNARDL	Small and large changes in climate policy uncertainty affect CO_2 emissions differently.	Shows that MTNARDL can provide more informative results than standard ARDL and NARDL models.

Table 2 indicates that the MTNARDL technique is a more general and flexible variant of the NARDL model. While the NARDL model introduced by Shin et al. (2014) enables us to investigate the asymmetric impacts of positive and negative shocks, the MTNARDL approach provided by Pal and Mitra (2015, 2016) takes into account both the direction and the magnitude of shocks. Uche and Effiom (2021), Asif et al. (2023) and Ben-Salha and Ayad (2025) find threshold-dependent responses of CO_2 emissions to macroeconomics and policy shocks. This methodological paradigm is the direct basis for the present study. This is because the effect of Brent oil prices on CO_2 emissions for Azerbaijan should not be measured as a general average effect rather it should be analyzed across low, medium and high price shock regimes.

3. METHODOLOGY

This study examines the relationship between oil prices (Brent) and CO_2 emissions in the context of oil price shocks-CO_2 emissions in Azerbaijan. As Azerbaijan is an oil-exporting and resource-dependent economy, the Brent oil price is considered as an external oil price shock that can affect CO_2 emissions through oil revenues, fiscal expansion, investment activity, energy use, and energy-intensive production. The primary objective is to evaluate whether the impact of changes in the Brent price on CO_2 emissions is linear and uniform or whether it differs across various regimes of oil price shocks.

The empirical baseline relationship is presented as follows:

$$\mathbf{LCO_{2t} = f(LBPP_t)} \qquad \textbf{(1)}$$

where LCO_{2t} is the logarithmic form of CO_2 emissions and $LBPP_t$ is the logarithmic form of Brent petroleum prices. The baseline model is presented in linear form as:

$$\mathbf{LCO_{2t} = \beta_0 + \beta_1 LBPP_t + u_t} \qquad \textbf{(2)}$$

Theoretically, the sign of $\boldsymbol{\beta_1}$ may depend on the economic and energy structure of the country. In oil-exporting economies, an increase in Brent prices may increase oil revenues, fiscal capacity, investment and energy-intensive economic activity, thereby raising CO_2 emissions. In this case, a positive coefficient is expected. However, higher oil prices may also encourage energy efficiency, technological modernization and transition toward alternative energy sources. Therefore, the magnitude and direction of the Brent price effect may vary depending on the size of the oil price shock.

To capture possible regime-dependent effects, the baseline model is extended into a multiple-threshold framework. Changes in LBPP are decomposed into four quartile-based regimes: Q_1**,** Q_2**,** Q_3 and Q_4. Q_1 represents negative Brent price shocks, Q_2 captures weak or transitional changes, Q_3 represents moderate positive Brent price shocks, and Q_4 captures large positive Brent price shocks. Accordingly, the regime-dependent specification can be written conceptually as:

$$\mathbf{LCO_{2t} = f[LBPP(Q_1)_t, LBPP(Q_2)_t, LBPP(Q_3)_t, LBPP(Q_4)_t]} \qquad \textbf{(3)}$$

This specification allows the effect of Brent petroleum prices on CO_2 emissions to be evaluated not as a single average effect, but separately across different oil price shock regimes.

The empirical analysis is carried out in several stages. First, the linearity of the Brent price–CO_2 emissions relationship is examined using the nonlinearity test proposed by Harvey, Leybourne and Xiao (2008). This test is appropriate because it allows the linearity hypothesis to be tested without imposing a strict prior assumption about whether the variables are I(0) or I(1). If the null hypothesis of linearity is rejected, the use of a nonlinear and regime-dependent modeling strategy becomes methodologically justified.

In the next stage, the stationarity properties of the variables are examined. The conventional ADF unit root test is first applied as a benchmark test (Dickey & Fuller, 1979, 1981). However, given that Azerbaijan's economy has experienced oil price shocks, financial fluctuations, exchange rate adjustments and other possible structural changes, relying only on the standard ADF test may be insufficient. Therefore, the Flexible Fourier ADF test is also employed to account for smooth structural changes without requiring the exact break dates to be known in advance (Enders & Lee, 2012). In addition, the Fractional Flexible Fourier ADF test is used because structural changes in macroeconomic and energy-related time series may not always follow integer-frequency cycles. This test allows Fourier frequencies to take fractional values and therefore provides a more flexible treatment of gradual and non-standard structural shifts (Omay, 2015).

After determining the integration properties of the variables, the effect of Brent price shocks on CO_2 emissions is estimated using the Multiple Threshold Nonlinear ARDL approach. The ARDL bounds testing framework is suitable because it allows the long-run relationship to be examined when the variables are a mixture of I(0) and I(1), provided that none of the variables is I(2). It is also appropriate for relatively small samples (Pesaran et al., 2001). While the NARDL model enables the asymmetric effects of positive and negative changes to be distinguished (Shin et al., 2014), it does not fully capture differences arising from the size of the shock. Since Brent prices are highly volatile, the magnitude of the shock may be as important as its direction. For this reason, this study employs the Multiple Threshold NARDL framework developed by Pal and Mitra (2015, 2016).

In the MTNARDL specification, LBPP changes are decomposed into four quartile regimes. This decomposition allows Brent price shocks to be evaluated across negative, weak, moderate positive and large positive regimes. The existence of a long-run relationship is tested using the PSS bounds testing approach, while finite-sample critical values are also considered due to the limited sample size (Kripfganz & Schneider, 2020). The error correction coefficient is used to evaluate the speed at which short-run deviations return to the long-run equilibrium. A negative and statistically significant error correction coefficient indicates that the long-run adjustment mechanism is valid.

To further examine the transmission path of Brent price shocks, dynamic multiplier effects are calculated. These multipliers show how the effect of each Brent price shock regime on CO_2 emissions evolves over time and how quickly it converges to its long-run level. In addition, Wald tests are applied to examine whether the effects of Brent price shocks are symmetric across regimes. The long-run asymmetry test evaluates whether the long-run coefficients of the Q_1–Q_4 regimes are equal, while the short-run asymmetry test examines whether short-run responses differ across regimes. Rejection of the symmetry hypothesis indicates that the Brent price–CO_2 emissions relationship is regime-dependent.

Overall, the econometric strategy allows the Brent price–CO_2 emissions nexus in Azerbaijan to be evaluated from several perspectives: nonlinearity, regime-dependent oil price shocks, long-run equilibrium, short-run adjustment, dynamic transmission and asymmetry across shock regimes.

4. DATA AND EMPIRICAL RESULTS

4.1. Data

This section introduces the data used in the empirical analysis. The study covers the period of 1990–2024 and investigates the relationship between CO_2 emissions and Brent petroleum prices in Azerbaijan. The dependent variable is CO_2 emissions and the main explanatory variable and external oil price shock is Brent petroleum price. Both variables are transformed into log form to reduce the differences in scale and to interpret the estimated coefficients in terms of elasticity. The definitions, transformations, data sources and expected effects of the variables are shown in Table 3.

Table 3. *Variable definitions and data sources*

Variable	Definition	Transformation	Source	Expected effect
CO_2	Carbon dioxide emissions	$LCO_2 = \ln(CO_2)$	World Bank (2026)	Dependent variable
BPP	Brent petroleum price	LBPP = ln(BPP)	Investing.com (2026)	Positive or regime-dependent

Table 3 shows the main variables of the study for the period 1990-2024. where LCO_2 denotes the log form of carbon dioxide emissions and LBPP denotes the log form of Brent petroleum prices. Given that Azerbaijan has an oil-exporting economy, it is expected that Brent oil price will affect CO_2 emissions through oil revenues, economic activity, energy demand and energy-intensive production. However, this effect may differ across oil price shock regimes. Thus, the expected effect of BPP is defined as positive or regime-dependent.

The descriptive properties of the variables are discussed before proceeding on to the econometric analysis. Descriptive statistics give preliminary information about the central tendency, variability, distributional shape and normality features of the series used in the model. Table 4 shows the mean, median, maximum, minimum, standard deviation, skewness, kurtosis, Jarque–Bera statistic and probability values for Brent petroleum prices and CO_2 emissions during the 1990–2024 period.

Table 4. *Descriptive Statistics*

	BPP	CO_2
Mean	52.99	3.956
Median	53.03	3.584
Maximum	111.57	7.821
Minimum	12.76	2.943
Std. Dev.	32.20	1.096
Skewness	0.401	2.535
Kurtosis	1.848	8.682
Jarque-Bera	2.873	84.57
Probability	0.238	0.000
Observations	35	35

Source: *author calculations*

As seen in Table 4, Brent petroleum prices vary significantly over the sample period. The mean value of BPP is 52.99, with minimum and maximum values of 12.76 and 111.57, respectively. The wide range is indicative of the high volatility of international oil prices from 1990 to 2024. The standard deviation of BPP is 32.20 which is quite high indicating the wide fluctuation around mean. The value of skewness is 0.401 which indicates a slightly right skewed distribution and the value of kurtosis is 1.848 which indicates a relatively flatter distribution as compared to normal distribution. The probability value of the Jarque-Bera test for BPP is 0.238, which does not reject the null hypothesis of normality.

For CO_2 emissions, the mean and median values are 3.956 and 3.584. The highest value is 7.821 and the lowest value is 2.943. CO_2 emissions have lower absolute variability compared to BPP, with a standard deviation of 1.096. However, the distributional properties of CO_2 are more pronounced. The value of skewness of 2.535 shows a high right-skewed distribution, which means that the relatively high emission values are clustered in some periods. The kurtosis value of 8.682 shows that the distribution is leptokurtic, with a sharper peak and heavier tails than normal distribution. The CO_2 series consistently reject the normality assumption as evidenced by the Jarque–Bera statistic.

Overall, the descriptive statistics indicate that the prices of Brent petroleum are highly volatile and the CO_2 emissions exhibit strong asymmetry and non-normal distributional features. Such preliminary results justify the use of nonlinear and regime-dependent econometric approaches as the relationship between Brent prices and CO_2 emissions might not be appropriately captured by a simple linear model.

In addition to descriptive statistics, a graphical investigation of the variables presents useful preliminary information about the time-series behavior of the variables. Figure 1 shows the developments of LCO_2 and LBPP for the time span of 1990–2024. This graphical analysis is useful to identify general trends, fluctuations and possible structural changes before proceeding to formal econometric tests.

As illustrated in Fig. 1, LCO_2 decreased in the early 1990s, then fluctuated moderately around a relatively stable level after the late 1990s. Some periods after 2010 can be seen to have a slightly upward tendency but the series does not have a continuously increasing trend. LBPP, on the other hand, shows much higher variation over the sample period. Brent petroleum prices increased sharply in the 2000s, reached a peak in the late 2000s and early 2010s, declined after 2014, and experienced renewed volatility in the post-2020 period.

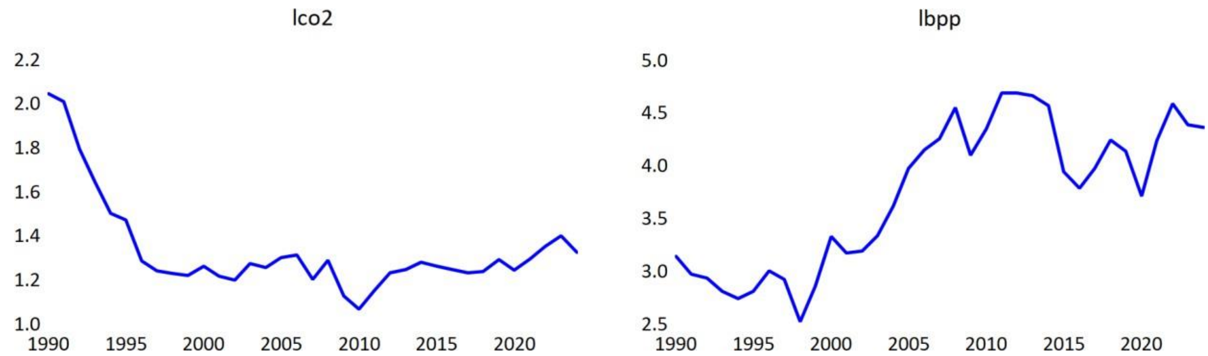

Figure 1. *Time-series plots of LCO_2 and LBPP*

Source: World Bank (2026) and Investing.com (2026).

Visual evidence suggests that Brent petroleum prices are much more volatile than CO_2 emissions. The different behaviour of the two series suggests the necessity of an econometric approach able to account for nonlinear and regime-dependent effects. In particular, the large spikes and dips in the LBPP indicate that the impact of Brent price movement on CO_2 emissions may differ among low, moderate and high oil price shock regimes.

4.2 Empirical Results

After an overview of the data structure, descriptive statistics and graphical behavior of the variables, this section presents the empirical results. The analysis begins with linearity tests, given that the knowledge about the variables presenting linear or non-linear behaviour is an important step before using threshold-based econometric models. If the null hypothesis of linearity is rejected, then the application of nonlinear and regime-dependent approaches such as MTNARDL is justified from a methodological point of view.

Table 5. *Linearity Test Results*

Variables	Harvey and Leybourne (2007) W*	Harvey et al. (2008) Wλ	Decision
LCO_2	26.99***	9.271***	Non-Linear
LBPP	27.00***	10.04***	Non-Linear

Note: The null hypothesis is linearity. For Harvey and Leybourne (2007), the W* statistic is evaluated with the $\chi^2(4)$ distribution, while for Harvey et al. (2008), the Wλ statistic is evaluated with the $\chi^2(2)$ distribution. *** indicates rejection of the null hypothesis of linearity at the 1% significance level.

The results of the linearity test for LCO_2 and LBPP are shown in Table 5. The W* statistics of Harvey and Leybourne (2007) are 26.99 for LCO_2 and 27.00 for LBPP, both of which are statistically significant at 1% level. Similarly, the W_λ statistics of Harvey et al. (2008) are 9.271 and 10.04 respectively and again suggest the rejection of null hypothesis of linearity. The results suggest that both variables are nonlinear.

The rejection of linearity provides strong methodological support for the use of nonlinear and regime-dependent econometric methods. Therefore, the MTNARDL approach is appropriate to explore whether the effects of Brent petroleum price shocks on CO_2 emissions are asymmetric across the shock regimes.

The unit root tests are performed following the examination of the deterministic trend properties of the variables. The reason a step is important because the specification of the unit root tests may depend on whether the series contains only an intercept or it contains both intercept and trend components. Therefore, linear and quadratic trend regressions were estimated for LCO_2 and LBPP. The results are summarized in Table 6.

Table 6. *Deterministic Trend Specification Results*

Variable	Model	Constant	Trend	$Trend^2$	Decision
LBPP	Linear trend	2.835***	0.054***	—	Trend is significant
LBPP	Quadratic trend	2.516***	0.113***	−0.0017**	Nonlinear trend component exists
LCO_2	Linear trend	1.539***	−0.011***	—	Trend is significant
LCO_2	Quadratic trend	1.849***	−0.068***	0.0017***	Nonlinear trend component exists

Note: ***, ** and * indicate statistical significance at the 1%, 5% and 10% levels, respectively.

As can be seen from table 6, both variables have statistically significant deterministic trend components. The linear trend specification gives a positive and significant trend coefficient for LBPP, implying that Brent petroleum prices followed an upward tendency during the sample period. The coefficient for LCO_2 for linear trend is negative and statistically significant, indicating a downward trend in CO_2 emissions over the sample period.

The quadratic trend results give additional information. For LBPP, the squared trend term is negati-ve and significant at the 5% level, which implies that the upward trend in Brent prices is not perfectly linear and tends to level off or decline in some periods. The squared trend term for LCO_2 is positive and significant at the 1% level, which indicates a nonlinear trend pattern with an initial decline followed by a partial recovery or stabilization over time.

These results indicate the need to estimate the unit root tests with intercept and trend specifications. Furthermore, the significance of the quadratic trend terms validates the application of Fourier-based unit root tests as the Flexible Fourier ADF and Fractional Fourier ADF tests are designed to capture smooth and nonlinear structural changes in time-series behavior.

Table 7. *Unit Root Test Results*

Variable	Form	ADF test statistic	FFADF statistic	Fourier frequency	FrFADF statistic	Fractional frequency	Decision
LCO_2	Level	-3.168	-3.1959	3	-2.531	0.2	Non-Stationary
LCO_2	1st diff	-5.460***	-	-	-	-	Stationary
LBPP	Level	-2.037	-3.089	1	-2.522	0.2	Non-Stationary
LBPP	1st diff	-5.542***	-	-	-	-	Stationary

Note: The null hypothesis is the presence of a unit root. ADF denotes the Augmented Dickey–Fuller test, FFADF denotes the Flexible Fourier ADF test, and FrFADF denotes the Fractional Flexible Fourier ADF test. ***, ** and * indicate rejection of the unit root null hypothesis at the 1%, 5% and 10% significance levels, respectively.

The results in Table 7 show the non-stationary of both LCO_2 and LBPP at level. While the ADF statistic of LCO_2 is significant at 5% level, the Fourier-based unit root tests do not provide consistent evidence of stationarity at level. Thus, under a conservative decision rule, LCO_2 is considered non-stationary in levels. Similarly, all reported tests indicate that LBPP is also non-stationary at level.

Both variables are stationary in first difference. The statistics of the first differences of LCO_2 and LBPP from the ADF test are significant at 1% level statistically. This implies that both variables are integrated of order one, I(1). The two variables are not I(2) so that the use of ARDL-type models, including the MTNARDL approach, is methodologically appropriate in the study of the long-run and short-run relationship between Brent petroleum prices and CO_2 emissions.

The selected MTNARDL model and its general specification are presented in advance estimation of the long-run and short-run coefficients. This step is important because it indicates the dependent variable, the decomposed explanatory variable, the threshold partitioning rule, lag selection criterion and overall model fit. The main features of the selected MTNARDL model are reported in Table 8.

Table 8. *MTNARDL Model Selection and General Characteristics*

Indicator	Result
Dependent variable	LCO_2
Explanatory variable	LBPP
Decomposition variable	LBPP
Partition type	Quartile
Number of regimes	4
Initial effective observations	33
Included observations	31
Maximum lag length	1
Information criterion	AIC
PSS case	Case 3
Selected ARDL specification	ARDL(1,1,0,0,0)
R-squared	0.7649
Adjusted R-squared	0.6287
Log-likelihood	56.4785
AIC	−88.9570
BIC	−71.7491
F-statistic	5.6187
F-statistic p-value	0.0005
RMSE	0.04998
Number of estimated models	16

The best model selected according to the AIC criterion is ARDL(1,1,0,0,0), and is listed in Table 8. LCO_2 is the dependent variable and LBPP is the main explanatory variable and decomposition variable. The quartile partition divides Brent price changes into four regimes, which allows the model to capture whether the impact of Brent petroleum price shocks on CO_2 emissions varies across shock magnitudes.

Model fit statistics indicate that the specification used fits quite well. The R-squared value of 0.7649 indicates that the model explains a good part of the variation of the LCO_2. The adjusted R-squared value is 0.6287 which is acceptable for a small sample. The F-statistic is significant at the 1 percent level indicating that the model is jointly significant. Further, the value of RMSE is low that means the model has lesser average error in prediction. Finally, the chosen MTNARDL specification is appropriate for the investigation of the regime-dependent link between Brent oil prices and CO_2 emissions in Azerbaijan.

The next step is to examine the distribution of LBPP change within the threshold regimes. As the MTNARDL approach allows the explanatory variable to be decomposed based on the magnitude of shocks, LBPP changes are divided into four quartile-based regimes. The quartile decomposition of LBPP changes is shown in Table 9.

Table 9. *Quartile Decomposition of LBPP Changes*

Regime	ΔLBPP interval	Observations	Share	Economic interpretation
Q1	[−0.2778; −0.0173]	8	25.0%	Negative Brent price shock regime
Q2	(−0.0173; 0.0394]	8	25.0%	Weak change / low shock regime
Q3	(0.0394; 0.0820]	8	25.0%	Moderate positive Brent price shock
Q4	(0.0820; 0.2852]	8	25.0%	Large positive Brent price shock

LBPP changes are evenly distributed in four quartile regimes as shown in Table 9. Each regime consists of 8 observations which is 25% of the sample. Q_1 captures negative changes in Brent prices, Q_2 captures weak or transitional ones. Q_3 and Q_4 correspond to a medium and large positive Brent price shock, respectively. Such a decomposition allows the model to test whether CO_2 emissions react differently to changes in Brent prices depending on the size and the sign of the shock. The quartile-based structure thus offers a suitable foundation for the estimation of the regime-dependent effects within the MTNARDL framework.

This long-run interpretation in the MTNARDL model is only valid in the presence of an error correction mechanism. A negative and statistically significant error correction coefficient indicates that deviations from equilibrium in the short-run are corrected over time and the system returns to its long-run path. Table 10 presents the speed of adjustment and the long-run coefficients of the Brent price shock regimes.

Table 10. *Speed of Adjustment and Long-run MTNARDL Coefficients*

Panel	Variable	Coefficient	Std. error	t/z-statistic	p-value	95% confidence interval
Speed of adjustment	LCO_2	−0.8332	0.1842	−4.5225	0.0002***	—
Long-run coefficient	LBPP(Q_1)	0.6417	0.2413	2.6594	0.0078***	[0.1688; 1.1146]
Long-run coefficient	LBPP(Q_2)	4.4537	1.4013	3.1782	0.0015***	[1.7072; 7.2001]
Long-run coefficient	LBPP(Q_3)	−0.3894	0.5143	−0.7571	0.4490	[−1.3975; 0.6187]
Long-run coefficient	LBPP(Q_4)	−0.0306	0.0968	−0.3160	0.7520	[−0.2203; 0.1591]

Note: ***, ** and * indicate statistical significance at the 1%, 5% and 10% levels, respectively.

The error correction coefficient in Table 10 is negative and statistically significant at the 1% level. The coefficient of −0.8332 means that about 83.3% of the short-run disequilibrium is corrected in the subsequent period. This result confirms the existence of a strong adjustment mechanism and supports the validity of the long-run relation between Brent petroleum price shocks and CO_2 emissions.

The long-run coefficients suggest that the effects of Brent price shocks are heterogeneous across regimes. The coefficients of LBPP(Q_1) and LBPP(Q_2) are positive and statistically significant at 1% level. In particular, the Q_2 coefficient is significantly larger than the other regimes, suggesting that weak or transitional Brent price changes are most strongly associated with CO_2 emissions in the long-run. On the other hand, the coefficients of LBPP(Q_3) and LBPP(Q_4) are negative but statistically insignificant. This implies that moderate and large positive shocks to the Brent price have no permanent effect on CO_2 emissions. Overall the results suggest the Brent price-CO_2 emissions relationship is regime-dependent rather than uniform to oil price shock levels.

After estimating the long-run coefficients, the short-run dynamics of the MTNARDL model were examined. The short-run coefficients indicate the extent to which changes in the price of Brent are transmitted to CO_2 emissions in the current and lagged periods. Table 11 presents the short-run results of the MTNARDL model.

As shown in Table 11, the short-run effects of Brent price shocks are mainly observed in the Q_3 and Q_4 regimes. The coefficient of D.LBPP(Q_3) is positive and statistically significant at 10% level, implying that moderate positive Brent price shocks increase CO_2 emissions in the short-run. The coefficient of D.LBPP(Q_4) is also positive and statistically significant at the 5% level, which suggests that large positive shocks in the Brent price have a short-run increasing effect on CO_2 emissions.

However, the coefficients for Q_1 and Q_2 are not statistically significant in the short-run. This could mean that negative and weak changes in Brent prices do not trigger an immediate and statistically significant response in CO_2 emissions. Considering the long-run results, the results reveal that weak and transitional Brent price changes are more relevant in the long-run relationship, while moderate and large positive Brent price shocks only affect CO_2 emissions in the short-run. This is consistent with the view that the Brent price-CO_2 emissions nexus is not only regime-dependent but also time-horizon dependent.

Table 11. *Short-run MTNARDL Coefficients*

Variable	Coefficient	Std. error	t-statistics	p-value
L1.D.LCO$_2$	0.0197	0.1767	0.1117	0.9122
D.LBPP(Q$_1$)	0.2510	0.6261	0.4010	0.6929
L1.D.LBPP(Q$_1$)	−0.3989	0.3126	−1.2761	0.2173
D.LBPP(Q$_2$)	1.5581	1.9355	0.8050	0.4308
D.LBPP(Q$_3$)	0.9322	0.4870	1.9143	0.0708*
D.LBPP(Q$_4$)	0.5297	0.2015	2.6282	0.0166**
Constant	1.5158	0.4340	3.4927	0.0024***

Note: ***, ** and * indicate statistical significance at the 1%, 5% and 10% levels, respectively.

The PSS (2001) bounds test was used to check the existence of long-run relationship in MTNARDL model. This test allows us to determine the existence of a cointegration relationship between shocks to the LCO$_2$ and Brent prices. Table 12 presents the results of cointegration test.

Table 12. *PSS Bounds Cointegration Test Results*

Test	Statistic	I(0)-I(1) 10%		I(0)-I(1) 5%		I(0)-I(1) 1%		p-value I(0)-I(1)		Decision
F-overall	6.534**	2.723	4.126	3.376	5.010	5.028	7.229	0.003	0.016	cointegration exists
t-dependent	−4.523**	−2.496	−3.623	−2.886	−4.087	−3.702	−5.060	0.002	0.025	
F-indep	9.386***	4.183	6.521	4.896	7.054	6.728	8.495	0.003	0.006	

The bounds test results of PSS(2001) for MTNARDL model are presented in Table 12. The F-overall statistic is 6.534, which is greater than the 5% upper-bound critical value. Similarly, the t-dependent statistic is more negative than the 5% upper-bound critical value. These results suggest that we reject the null hypothesis of no long-run relationship.

This is supported by the F-independent statistic as well. Its value of 9.386 exceeds the 1% upper-bound critical value, implying a strong evidence of long-run relationship among the variables. The PSS bounds test results indicate that cointegration exists between CO_2 emissions and Brent petroleum price shock regimes in the long-run . Hence, the long-run coefficients obtained from the MTNARDL model are interpretable.

To check the cointegration and coefficient results, we tested the model's diagnostic properties. To this end, tests of normality, serial correlation, heteroscedasticity, functional form and parameter stability were performed. Table 13 presents the results of the diagnostic tests.

Table 13 shows the results of the diagnostic tests for the chosen MTNARDL model. The null hypothesis of normality cannot be rejected, which implies that the residuals are normally distributed, as suggested by the Jarque–Bera test. The Breusch-Godfrey LM test with four lags does not show any evidence of serial correlation that the residuals are not suffering from autocorrelation up to lag 4. Furthermore, the White test indicates that there is no heteroskedasticity and the Ramsey RESET test indicates that the functional form of the model is correctly specified.

Table 13. *Diagnostic Test Results of the MTNARDL Model*

Test group	Test	Statistic	p-value	Decision
Normality	Jarque–Bera	2.6601	0.2645	Normality cannot be rejected
Serial correlation	BG LM(4)	6.1990	0.1848	No evidence of serial correlation up to lag 4
Heteroskedasticity	White (NR^2)	2.1743	0.3372	No evidence of heteroskedasticity
Functional form	Ramsey RESET	0.4985	0.6873	Functional form is correctly specified
Stability	CUSUM	Stable	—	Parameters are stable
Stability	CUSUM-SQ	Stable	—	Parameters are stable

The stability of the estimated parameters is also confirmed by the CUSUM and CUSUM-SQ tests. The two tests indicate that the model parameters are constant over the sample period . Overall, the diagnostic test results indicate that the selected MTNARDL specification is statistically reliable and suitable for the interpretation of the long-run and short-run effects of Brent petroleum price shocks on CO_2 emissions.

The overall results of the MTNARDL reveal that the effect of Brent petroleum prices on CO_2 emissions is non-linear and different across different shock regimes. The long-run results show that the statistically significant effects are mainly in the Q_1 and Q_2 regimes. This indicates that the negative Brent price changes and weak or transitional price movements are more closely associated with the long-run equilibrium behavior of CO_2 emissions in Azerbaijan.

The short-run results, however, indicate that the significant effects are concentrated in the regimes Q_3 and Q_4. These regimes correspond to moderate and large positive shocks to the Brent price. Thus, the effect of increases in Brent prices on CO_2 emissions is observed only in the short-run when the price shock is relatively large. Such shocks may temporarily boost economic activity, energy demand, transportation, construction and other energy-intensive activities but their effects do not persist as statistically significant long-run impacts.

In summary, these results imply that the Brent price–CO_2 emissions nexus in Azerbaijan is dependent on the magnitude of the oil price shock and the time horizon. Weak and transitional price changes are more relevant for the long-run relationship. Moderate and large positive price shocks induce more immediate short-run responses.

Table 14. *Dynamic and Cumulative Multiplier Results for Brent Price Shocks*

Horizon	Q_1 Dynamic	Q_1 Cumulative	Q_2 Dynamic	Q_2 Cumulative	Q_3 Dynamic	Q_3 Cumulative	Q_4 Dynamic	Q_4 Cumulative
0	0.251	0.251	1.558	1.558	0.932	0.932	0.530	0.530
1	-0.069	0.183	2.443	4.001	-1.083	-0.151	-0.456	0.073
2	0.381	0.564	0.425	4.427	-0.220	-0.371	-0.096	-0.022
3	0.073	0.636	0.031	4.457	-0.020	-0.391	-0.009	-0.031
4	0.006	0.642	-0.003	4.455	0.001	-0.390	0.000	-0.031
5	-0.000	0.642	-0.001	4.454	0.001	-0.389	0.000	-0.031
10	0.000	0.642	0.000	4.454	0.000	-0.389	0.000	-0.031
15	0.000	0.642	0.000	4.454	0.000	-0.389	0.000	-0.031
20	0.000	0.642	0.000	4.454	0.000	-0.389	0.000	-0.031
LRM	—	0.642	—	4.454	—	-0.389	—	-0.031

Table 14 presents the dynamic and cumulative multiplier results for Brent price shock regimes. The results show that the adjustment paths differ across Q_1–Q_4 regimes. In the Q_1 regime, the cumulative multiplier gradually converges to a positive long-run multiplier of 0.642. The strongest long-run response is observed in the Q_2 regime, where the cumulative effect quickly rises and stabilizes around 4.454. This indicates that weak or transitional Brent price changes have the largest long-run association with CO_2 emissions.

For Q_3 and Q_4, the initial dynamic responses are positive, but the cumulative effects turn negative after the first periods and converge to −0.389 and −0.031, respectively. This suggests that moderate and large positive Brent price shocks may generate short-run responses, but their long-run cumulative effects are weak and negative. Overall, the multiplier results confirm that the transmission of Brent petroleum price shocks to CO_2 emissions is regime-dependent and differs across time horizons.

The Q_2 regime is the regime where the cumulative effect of Brent price shocks on CO_2 emissions is the strongest as shown in Figure 2. The multiplier for Q_2 rises in the short term and settles at around 4.45. In the Q_1 regime the effect is positive but much weaker than in Q_2. For the Q_3 and Q_4 regimes, the first period displays a positive reaction, but the effect is not persistent in the subsequent horizons, converging to a negative long-run level. This finding suggests that the effect of Brent price shocks on CO_2 emissions is regime-dependent not only in terms of direction but also in terms of transmission dynamics over time.

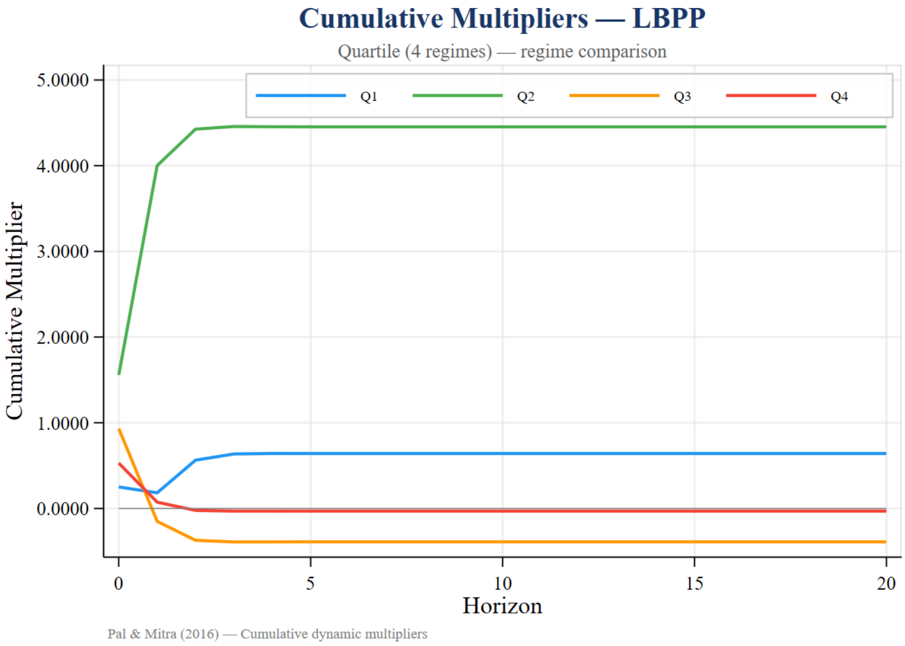

Figure 2. *Cumulative multiplier results for Brent price shocks*

Following the dynamic multiplier analysis, we looked at the speed of adjustment and persistence profile to check the speed with which the system reverts to its long-run equilibrium after a shock. These indicators show how quickly disequilibrium is corrected and how long the effect of a shock lasts. The error correction coefficient, half-life measures and persistence profile are reported in Table 15.

Table 15. *Speed of Adjustment and Persistence Profile Results*

Panel	Indicator	Results
Speed of adjustment	ECM coefficient (alpha)	−0.8332
Speed of adjustment	Half-life	0.3870
Speed of adjustment	50% adjustment time	0.3870
Speed of adjustment	90% adjustment time	1.2857
Speed of adjustment	99% adjustment time	2.5713
Persistence profile	Profile half-life	1 period

The adjustment to long-run equilibrium is strong and fast as shown by the ECM coefficient of -0.833 in Table 15. This means that around 83.3% of the disequilibrium between Brent petroleum price shocks and CO_2 emissions is corrected in the following period. The half-life value of 0.387 indicates that the half of the shock effect disappears in less than one period.

This interpretation is also consistent with the adjustment time indicators. Following the 1.286 periods the adjustment to the disequilibrium is about 90% complete and after 2.571 periods it is about 99% complete. The half-life of the persistence profile of one period suggests that shocks do not have a long-lasting destabilizing impact on the system. Overall, these results indicate that the impact of Brent price shocks on CO_2 emissions differs across regimes, but the system reverts to its long-run equilibrium relatively quickly.

One of the main advantages of the MTNARDL approach is that it allows testing for different effects of shocks across regimes; For this purpose we performed long-run and short-run asymmetric Wald tests. The results of the Wald test for symmetry of Brent price shocks across regimes are reported in Table 16.

***Table 16.** Wald Asymmetry Test Results for Brent Price Shocks*

Test	Null hypothesis	Statistic	p-value	Decision
Long-run symmetry	H_0: LR Q_1 = … = LR Q_4	$\chi^2(3) = 18.19$	0.0004***	Reject H_0
Q_1 vs Q_4 long-run difference	Long-run effects of Q_1 and Q_4 are equal	z = −2.559	0.0105**	Reject H_0
Short-run symmetry	Short-run effects of Q_1 and Q_4 are equal	F = 0.195	0.6638	Do not reject H_0

Note: ***, ** and * indicate statistical significance at the 1%, 5% and 10% levels, respectively.

Table 16 provides strong evidence of long-run asymmetry in the Brent price shock regimes. The long-run symmetry test is statistically significant at 1% level which indicates the long-run effects of Q_1, Q_2, Q_3 and Q_4 are not equal. This confirms that Brent price shocks have different effects on CO_2 emissions depending on the regime of the shock.

This conclusion is also supported by the pairwise comparison between Q_1 and Q_4. We reject the null hypothesis of equal long-run effects of Q_1 and Q_4 at 5% level. This means that statistically different long-run effects of negative and large positive Brent price shocks are found on CO_2 emissions.

However, the short-run symmetry test is statistically insignificant . This indicates that there is no strong evidence of short-run asymmetry between Q_1 and Q_4. Thus, the asymmetric effect of Brent price shocks seems to be a long-run phenomenon rather than a short-run one.

Although the Wald tests suggest overall asymmetry, the pairwise long-run difference and asymmetric ratio indicators were further examined to identify the direction and magnitude of the difference between the regimes. These results show the regimes that are more resilient and the regimes that are more sensitive to Brent price shocks. The inter-regime long-run differences and asymmetry measures are reported in Table 17.

Although the Wald tests confirm the presence of long-run asymmetry, they do not show which regimes differ from each other or how strong the asymmetry is. Therefore, pairwise long-run differences and asymmetric ratio indicators were also examined. Table 17 presents the regime-specific long-run differences and asymmetry measures.

***Table 17.** Pairwise Long-run Differences and Asymmetry Indicators Across Regimes*

Panel	Comparison / Regime	Result	Std. error	z-statistic	p-value	Interpretation
Asymmetric ratio	Max(Q_2)/Min(Q_3)	11.437	—	—	—	Strong asymmetry
Asymmetric ratio	|LR(Q_2)/LR(Q_1)|	6.941	—	—	—	$Q_2 > Q_1$
Asymmetric ratio	|LR(Q_3)/LR(Q_2)|	0.087	—	—	—	$Q_3 < Q_2$
Asymmetric ratio	|LR(Q_4)/LR(Q_3)|	0.079	—	—	—	$Q_4 < Q_3$
LR range	[Min; Max]	[−0.389;4.454]	—	—	—	Spread = 4.843
Pairwise difference	Q_1 vs Q_2	3.812	1.373	2.777	0.0055***	Significant difference
Pairwise difference	Q_1 vs Q_3	−1.031	0.554	−1.862	0.0626*	Significant at the 10% level
Pairwise difference	Q_1 vs Q_4	−0.672	0.263	−2.559	0.0105**	Significant difference
Pairwise difference	Q_2 vs Q_3	−4.843	1.819	−2.663	0.0078***	Significant difference
Pairwise difference	Q_2 vs Q_4	−4.484	1.354	−3.313	0.0009***	Significant difference
Pairwise difference	Q_3 vs Q_4	0.359	0.604	0.594	0.5524	Not significant

Note: ***, ** and * indicate statistical significance at the 1%, 5% and 10% levels, respectively. LR denotes the long-run coefficient.

Table 17 provides additional evidence on the long-run effect of Brent price shocks on CO_2 emissions that varies substantially across regimes. The asymmetric ratio indicators show that the strongest long-run response is in the Q_2 regime, while the weakest responses are in Q_3 and Q_4. Specifically, the long-run effect in Q_2 is approximately 6.94 times larger than the long-run effect in Q_1, which indicates that weak or transitional Brent price changes are much more strongly associated with CO_2 emissions in the long-run.

The pairwise comparisons also give evidence of regime-dependent long-run effects. The differences between Q_1 and Q_2, Q_1 and Q_4, Q_2 and Q_3 and Q_2 and Q_4 are statistically significant. Only the difference between Q_1 and Q_3 is significant at the 10% level and the difference between Q_3 and Q4 is not statistically significant. This indicates that the main source of long-run asymmetry is the difference between the Q_2 regime and the other regimes.

Overall, the results confirm the heterogeneous effect of Brent price shocks on CO_2 emissions. The effect in the long-run is the most significant in the Q_2 regime. Moderate and large positive shocks represented by Q_3 and Q_4 do not differ significantly in the long-run. This result supports the conclusion that the relationship between Brent price and CO_2 emissions in Azerbaijan is highly dependent on the regime.

Figure 3 reports the long-run coefficients of Brent price shocks across the four quartile regimes to provide a visual summary of the regime-specific long-run effects. This figure is consistent with the results of the pairwise comparison in Table 17 and is able to visually capture the direction and the size of the long-run multipliers.

As shown in Figure 3, there are large differences in the long-run coefficients across regimes. The Q_2 regime has the highest positive long-run multiplier, implying that weak or transitional changes in Brent price have the strongest long-run association with CO_2 emissions. The coefficient of the Q_1 regime is also positive, but the magnitude is much smaller than that of Q_2.

By contrast, the long-run coefficients for the Q3 and Q4 regimes are negative. However, the magnitudes of these coefficients are small, especially in the Q4 regime. This indicates that moderate and large positive shocks to Brent prices do not have strong positive long-run effects on CO_2 emissions. Instead, their impact appears to be more short-run, as already indicated by the short-run MTNARDL results.

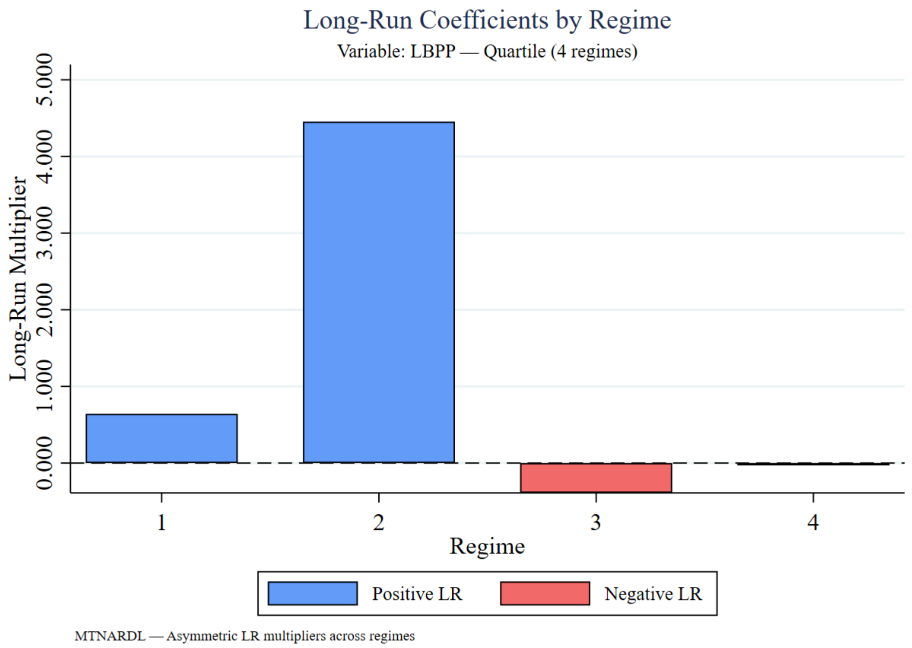

Figure 3. *Long-run Coefficients by Brent Price Shock Regimes*

Overall, Figure 3 supports the main finding of high regime dependence of the Brent price-CO_2 emissions nexus in Azerbaijan. This effect is not the same for all Brent price shocks. The strongest long-run response is found in the Q_2 regime, whereas large positive shocks do not lead to long-run persistent increases in CO_2 emissions.

5. DISCUSSION OF RESULTS

The empirical analysis starts with preliminary tests to evaluate the relevance of nonlinear and threshold-based modelling. The null of linearity for both LCO_2 and LBPP is rejected by the linearity tests of Harvey and Leybourne (2007) and Harvey et al. (2008). The result indicates that the variables follow a nonlinear pattern and provides methodological support for the use of a nonlinear and regime-dependent framework.

Deterministic trend regressions were estimated prior to unit root analysis. The results indicate statistically significant trend components for both LCO_2 and LBPP. The quadratic trend terms are also significant indicating that the variables do not follow purely linear deterministic paths. This result further supports the use of Fourier-based unit root tests that are designed to capture smooth and nonlinear structural changes in the time-series behavior.

The results of the unit root tests show that the two variables are non-stationary in levels but become stationary after first differencing. The standard ADF test indicates some evidence of stationarity for LCO_2 at level but this is not supported consistently by the Fourier-based unit root tests. Thus, under a conservative decision rule, both LCO_2 and LBPP are taken as I(1). This is methodologically correct as neither of the variables is integrated of order two and ARDL-type models including MTNARDL are appropriate.

The chosen specification of MTNARDL is ARDL(1,1,0,0,0) based on the AIC criterion. LBPP is divided into four regimes based on quartiles. Q_1 is negative Brent price shocks, Q_2 is weak or transitional changes, Q_3 is moderate positive Brent price shocks, and Q_4 is large positive Brent price shocks. With such a decomposition one can assess the impact of changes in Brent prices on CO_2 emissions for different shock sizes rather than an average effect.

The long-run results indicate that the error correction coefficient is −0.833 and is statistically significant at 1% level. This implies that about 83.3% of the short-run disequilibrium is corrected in the next period. Thus the model suggests that speed of adjustment to long-run equilibrium is strong and rapid. The PSS bounds test further affirms the existence of a long-run relationship between CO_2 emissions and Brent price shock regimes .

The long-run coefficients display a distinct regime-dependent pattern. The long-run effects of Q_1 and Q_2 regimes are positive and statistically significant whereas Q_3 and Q_4 regimes are statistically insignificant in the long-run. The Q_2 regime demonstrates the largest long-run response with the coefficient larger in absolute value than the coefficients in the other regimes. This suggests that the weak or transitionary changes in Brent prices are more closely related to the long-run equilibrium behavior of CO_2 emissions in Azerbaijan.

The short-run results reveal a different pattern. The important short-run effects are confined to the Q_3 and Q_4 regimes. These regimes are characterized by moderate and large positive Brent price shocks. The positive and statistically significant short-run coefficients suggest that increases in Brent prices could lead to immediate reactions in CO_2 emissions, potentially via temporary increases in economic activity, energy demand, transportation, construction and other energy-intensive activities. However, these effects are not statistically significant long-run impacts.

The results of dynamic multipliers show that the transmission of Brent price shocks differs between regimes and time horizons. The Q_2 cumulative multiplier increases sharply and converges to the largest long-run multiplier. The Q_1 multiplier converges to a smaller positive long-run multiplier. In contrast, the initial dynamic responses of Q_3 and Q_4 are positive but the cumulative effects turn negative later and stabilize at weak long-run levels. This supports the conclusion that the nexus between Brent oil price and CO_2 emissions is not homogeneous across regimes of oil price shocks.

The Wald asymmetry tests offer further evidence of regime dependence. We reject the long-run symmetry hypothesis at the 1% level, indicating that long-run effects of Q_1–Q_4 regimes are statistically different. Pairwise comparison of Q_1 and Q_4 also shows a statistically significant long-run difference. However, the short-run symmetry test is not significant that indicates the main asymmetry is more obviously observed in the long-run structure.

The diagnostic tests suggest that the chosen MTNARDL model is generally valid. Residuals are normally distributed, no evidence of heteroskedasticity, the functional form is correctly specified and the CUSUM and CUSUM-SQ tests are also confirming the stability of parameters. The Breusch–Godfrey LM test with four lags does not suggest serial correlation. The diagnostic results support the overall validity of the estimated MTNARDL model.

Overall, the empirical results show that the Brent petroleum price shocks affect the CO_2 emissions in Azerbaijan in a time horizon and regime-dependent way. The long-run relationship is more relevant for negative and weak Brent price changes, while the short-run responses are more contemporaneous for moderate and large positive shocks.

6. POLICY RECOMMENDATIONS AND CONCLUSION

The empirical results indicate that the impact of Brent oil price shocks on CO_2 emissions in Azerbaijan is regime-dependent and time-horizon-dependent. Therefore, environmental and energy policies should not treat changes in oil prices as if they had a uniform effect. Policy responses should be calibrated to the magnitude of oil price shocks and their expected short- and long-term consequences.

First, short-run environmental monitoring is required in periods of moderate and large positive Brent price shocks. The results indicate that Q_3 and Q_4 regimes are mostly relevant for CO_2 emissions in the short-run. That means that high spikes in oil prices can temporarily increase economic activity, energy demand, transportation, building and other energy-consuming sectors. Policymakers need to strengthen short-term monitoring of emissions, enhance energy efficiency controls and prevent temporary increases in oil revenues from leading to increased carbon-intensive activities during these periods.

Second, we should not ignore Brent price changes that are weak and transitional. The long-run results show that Q_1 and especially Q_2 regimes are more associated with the long-run equilibrium behavior of CO_2 emissions. This suggests that even small or gradual oil price changes can impact emissions through long-lived macroeconomic and energy-use channels. Environmental policy should therefore not only be concerned with large oil price shocks, but also with the cumulative long-run effects of smaller oil price changes.

Third, oil revenues must be used for low-carbon investment. Higher oil prices may create additional fiscal space, and these additional revenues could be used to support renewable energy projects, energy efficiency programs, low-carbon infrastructure and cleaner technologies. This would reduce the risk that rising oil prices reinforce fossil fuel dependence and carbon-intensive modes of production.

Fourth, Azerbaijan should improve the policies to reduce carbon lock-in. High oil revenues could make conventional fossil fuel development more appealing. Public investment decisions should prioritize diversification, the scaling up of renewable energy, the upgrading of industrial technologies and sustainable transport systems to avoid this. Such policies would help to prevent oil price spikes from slowing down the energy transition process.

Fifth, energy efficiency policies should be connected to oil price cycles. Brent prices can go up sharply and in the short-run this can allow energy-intensive sectors to grow. Therefore, stricter efficiency standards, cleaner production requirements and targeted incentives for low-emission technologies should be introduced or strengthened during oil price booms. This would help ease the short-run emission pressure identified in the Q_3 and Q_4 regimes.

Finally, the oil price risk should be incorporated into climate and energy planning in Azerbaijan. The results show that the environmental effects of Brent price shocks depend on the magnitude of the shock and the time horizon. Oil price scenarios and risk assessments based on regimes should thus be included in climate policy. This would enable policy makers to design more flexible environmental policies which react differently to weak, moderate and large oil price shocks.

In general, the findings suggest that Azerbaijan's environmental policy should not focus solely on average oil price effects but should also be regime-sensitive. A policy framework that combines short-run emission control during large oil price shocks with long-run investment in renewable energy and energy efficiency would be more effective in reducing environmental risks associated with oil price volatility.

Author Contributions: Conceptualization, N.G.; methodology, N.G.; data curation, N.G.; formal analysis, N.G.; writing—original draft preparation, N.G. and N.Z.; writing of the introduction and literature review, N.Z.; writing—review and editing, N.G. and F.M.; supervision, F.M. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding. The authors confirm that no financial support was received from any institution, organization or funding body for the preparation of this study.

Conflicts of Interest: The authors declare no conflict of interest.

REFERENCES

Agbanike, T. F., Nwani, C., Uwazie, U. I., Anochiwa, L. I., Onoja, T.-G. C., & Ogbonnaya, I. O. (2019). Oil price, energy consumption and carbon dioxide (CO_2) emissions: Insight into sustainability challenges in Venezuela. *Latin American Economic Review, 28*, Article 8. https://doi.org/10.1186/s40503-019-0070-8

Aliyev, F., Eylasov, N., Gasim, N., & Şahinler, A. N. (2024). Impact of Nuclear Energy Consumption on CO2 Emissions in South Korea: Evidence from Fourier Bootstrap ARDL Bound Test. *Journal of Sustainable Development Issues*, *2*(1), 51-66. https://doi.org/10.62433/josdi.v2i1.23

Asif, M., Sharma, V., Sharma, H. P., Aldawsari, H., Wani, S. K., Khosla, S., & Chandniwala, V. J. (2023). Is fiscal deficit 'curse' or 'haven' for environmental quality in India? Empirical investigation employing battery of distinct ARDL approaches. *Heliyon, 9*(10), e20711. https://doi.org/10.1016/j.heliyon.2023.e20711

Ayyubova, N. S. (2023). Analysis of the impact of global oil prices on GDP: On the example of the Azerbaijan Republic. *Statistics and Economics, 20*(2), 21–40. https://doi.org/10.21686/2500-3925-2023-2-21-40

Ben-Salha, O., & Ayad, H. (2025). Does climate policy uncertainty matter for sectoral environmental sustainability? Fresh evidence from the multiple threshold nonlinear ARDL model. *Stochastic Environmental Research and Risk Assessment, 39*(5), 1787–1804. https://doi.org/10.1007/s00477-025-02928-y

Dickey, D. A., & Fuller, W. A. (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association, 74*(366a), 427–431. https://doi.org/10.1080/01621459.1979.10482531

Dickey, D. A., & Fuller, W. A. (1981). Likelihood ratio statistics for autoregressive time series with a unit root. *Econometrica, 49*(4), 1057–1072. https://doi.org/10.2307/1912517

Enders, W., & Lee, J. (2012). The flexible Fourier form and Dickey–Fuller type unit root tests. *Economics Letters, 117*(1), 196–199. https://doi.org/10.1016/j.econlet.2012.04.081

Harvey, D. I., Leybourne, S. J., & Xiao, B. (2008). A powerful test for linearity when the order of integration is unknown. *Studies in Nonlinear Dynamics & Econometrics, 12*(3), Article 2. https://doi.org/10.2202/1558-3708.1582

Hasanov, F. J., Mukhtarov, S., & Suleymanov, E. (2023). The role of renewable energy and total factor productivity in reducing CO_2 emissions in Azerbaijan: Fresh insights from a new theoretical framework coupled with Autometrics. *Energy Strategy Reviews, 47*, 101079. https://doi.org/10.1016/j.esr.2023.101079

Investing.com, (2026). https://www.investing.com/commodities/brent-oil-historical-data. Accessed on 05.04.2026.

Kartal, M.T., Taşkın, D. & Depren, S.K. (2024). Interrelationship between environmental degradation, geopolitical risk, crude oil prices, and green bonds: Evidence from the globe by sectoral analysis, *Gondwana Research,* 132, 2024, 249-258, https://doi.org/10.1016/j.gr.2024.05.004.

Kartal, M.T., Magazzino, C., Taşkın, D., Depren, Ö. Ayhan, F. (2025). Efficiency of green bond, clean energy, oil price, and geopolitical risk on sectoral decarbonization: Evidence from the globe by daily data and marginal effect analysis, *Applied Energy*, 392, 125963, https://doi.org/10.1016/j.apenergy.2025.125963.

Kripfganz, S., & Schneider, D. C. (2020). Response surface regressions for critical value bounds and approximate p-values in equilibrium correction models. *Oxford Bulletin of Economics and Statistics, 82*(6), 1456–1481. https://doi.org/10.1111/obes.12377

Mikayilov, J. I., Mukhtarov, S., Mammadov, J., & Aliyev, S. (2020). Environmental consequences of tourism: Do oil-exporting countries import more CO_2 emissions? *Energy Sources, Part B: Economics, Planning, and Policy, 15*, 172–185. https://doi.org/10.1080/15567249.2020.1753856

Mukhtarov, S., Aliyev, F., Aliyev, J., & Ajayi, R. (2023). Renewable energy consumption and carbon emissions: Evidence from an oil-rich economy. *Sustainability, 15*(1), 134. https://doi.org/10.3390/su15010134

Mukhtarov, S., Mikayilov, J. I., Maharramov, S., Aliyev, J., & Suleymanov, E. (2022). Higher oil prices, are they good or bad for renewable energy consumption: The case of Iran? *Renewable Energy, 186*, 411–419. https://doi.org/10.1016/j.renene.2021.12.135

Mukhtarov, S., Kartal, M. T., & Ayhan, F. (2026). The environmental impact of AI-driven innovation: Evidence from the United States. Sustainable Welfare, 1, 33–42. https://doi.org/10.58225/sw.2026.1-33-42

Nwani, C. (2017). Causal relationship between crude oil price, energy consumption and carbon dioxide (CO_2) emissions in Ecuador. *OPEC Energy Review, 41*(3), 201–225. https://doi.org/10.1111/opec.12102

Omay, T. (2015). Fractional frequency flexible Fourier form to approximate smooth breaks in unit root testing. *Economics Letters, 134*, 123–126. https://doi.org/10.1016/j.econlet.2015.07.010

Pal, D., & Mitra, S. K. (2015). Asymmetric impact of crude price on oil product pricing in the United States: An application of multiple threshold nonlinear autoregressive distributed lag model. *Economic Modelling, 51*, 436–443.

Pal, D., & Mitra, S. K. (2016). Asymmetric oil product pricing in India: Evidence from a multiple threshold nonlinear ARDL model. *Economic Modelling, 59*, 314–328.

Pesaran, M. H., Shin, Y., & Smith, R. J. (2001). Bounds testing approaches to the analysis of level relationships. *Journal of Applied Econometrics, 16*(3), 289–326. https://doi.org/10.1002/jae.616

Şahinler, A. N. (2025). The impact of carbon allowance and oil prices on low-carbon footprint stocks: Evidence from RALS cointegration analysis. *Journal of Sustainable Development Issues, 3*(1), 22–31. https://doi.org/10.62433/josdi.v3i1.50

Shin, Y., Yu, B., & Greenwood-Nimmo, M. (2014). Modelling asymmetric cointegration and dynamic multipliers in a nonlinear ARDL framework. In R. C. Sickles & W. C. Horrace (Eds.), *Festschrift in honor of Peter Schmidt: Econometric methods and applications* (pp. 281–314). Springer. https://doi.org/10.1007/978-1-4899-8008-3_9

Toda, H. Y., & Yamamoto, T. (1995). Statistical inference in vector autoregressions with possibly integrated processes. Journal of Econometrics, 66(1–2), 225–250. https://doi.org/10.1016/0304-4076(94)01616-8

Uche, E., & Effiom, L. (2021). Financial development and environmental sustainability in Nigeria: Fresh insights from multiple threshold nonlinear ARDL model. *Environmental Science and Pollution Research, 28*, 39524–39539. https://doi.org/10.1007/s11356-021-12843-8

World Bank, (2026). World Development Indicators. https://databank.worldbank.org/source/world-development-indicators#. Accessed on 05.04.2026.